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Rapid Note

Resonance split of ballistic conductance peaks in electric and magnetic superlattices

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Abstract. Resonant peak splitting for ballistic conductance in finite electric superlattices (ES) and magnetic superlattices (MS) was investigated theoretically. It is shown that, for electron tunneling through the ES (MS) of identical n electric (magnetic) barriers, the resonance split of the conductance peak is $(n-1)$-fold; while for electron tunneling through the ES (MS) made of two different barriers, one resonant window of the former splits into two subwindows, within each of which the resonance split is $(m-1)$-fold, where m is the number of the renormalized building blocks consisting of two different barriers of the latter.

PACS. 73.40.Gk Tunneling – 73.61.-r Electrical properties of specific thin films and layer structures (multilayers, superlattices, quantum wells, wires, and dots) – 03.65.Ge Solutions of wave equations: bound states

Electronic transport in a two-dimensional electron gas (2DEG) has received much attention since von Klitzing *et al.* discovered the quantum Hall effect [1]. High mobility 2DEG formed in GaAs/AlGaAs semiconductor heterostructures allows electrons to move ballistically over distances of the order of several microns, which facilitates the experimental study of the ballistic transport of electrons [2]. On the other hand, it allows us to investigate various effects of artificial potential that can be realized, for example, by placing a microstructured gate electrode or ferromagnetic or superconductor on the surface. The resultant simple type of potential is a spacially modulated electrostatic or magnetic potential with nanoscale period [3]. It gives rise to oscillatory magnetoresistance known as Weiss oscillation [4]. In addition, due to the quantum confining effects induced by magnetic confinement, renewed interest has been paid to the resonant tunneling through the multiple-barrier magnetic structure [5].

It is well known that $(n-1)$ resonant peaks exist in the transmission as a function of incidence energy for electron tunneling through identical-n-barrier semiconductor superlattices [6]. Very recently we demonstrated that such resonant peak splitting also exists in the ballistic conductance for electron tunneling through the finite MS of identical magnetic barriers [7], though electron tunneling through magnetic structure is inherently a two-dimensional process [5]. In 1998 Guo *et al.* investigated the resonant peak splitting for transmission in the semiconductor and magnetic superlattices by periodically juxtaposing two different barriers [8]. They found that one resonant domain in the identical-barrier superlattice splits into two subdomains in the different-barrier superlattice case.

In this letter, we studied resonant peak splitting for ballistic conductance in two types of electric and magnetic supurlattices. One is the periodic arrangement of identical electric (magnetic) barriers, while the other is periodically juxtaposed with two different electric (magnetic) barriers. Since the resonance split of transmission and ballistic conductance does not depend on the actual shape of the electric (magnetic) barrier in ES (MS) [5–8], we choose the rectangular barrier (barriers) as the building block for ES (MS). We show that, although electron tunneling in ES is a one-dimensional process while that in MS is inherently two-dimensional, the resonant peak splitting rule for their ballistic conductance is the same. Therefore, one can depict the resonance split for electron tunneling through ES and through MS in a unified way.

We consider 2DEG electron tunneling through a periodic electrostatic potential shown in Figure 1a. ES is formed by periodically arranging two building blocks A and B. Each block consists of one electric barrier (with height U_{A} or U_{B} and width l_{EA} or l_{EB}) and one potential well of width l_{E}. For perfect identical-barrier ES, A = B. In the single electron approximation, the corresponding Schrödinger equation for electron tunneling through

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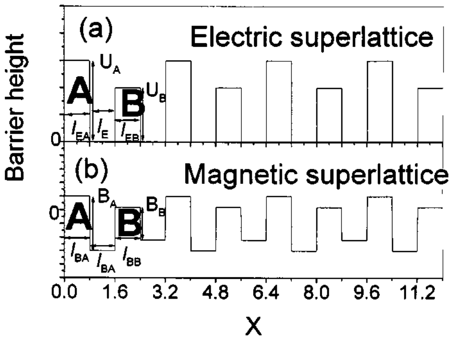

Fig. 1. The finite superlattice potential experienced by a 2DEG electrons: (a) ES constructed by periodically arranging two different-height electric barriers A and B, (b) MS made of two different magnetic barriers A and B.

the finite ES reads

$$\left[\frac{1}{2m^*}(p_x^2+p_y^2)+U(x)\right]\Psi(x,y)=E\Psi(x,y), \quad (1)$$

where m^* is the effective electron mass and $U(x)$ is the potential of the ES. Now we consider electron motion in the 2DEG subjected to a perpendicular periodic magnetic field (along the z direction) as shown in Figure 1b. MS is similarly obtained by periodically arranging two magnetic building blocks A and B. Each block consists of one magnetic barrier (with height $B_{\rm A}$ or $B_{\rm B}$ and width $l_{\rm BA}$ or $l_{\rm BB}$), and one magnetic well (with height $-B_{\rm A}$ or $-B_{\rm B}$ and width $l_{\rm BA}$ or $l_{\rm BB}$). The Schrödinger equation governing the electron motion in the MS can be written as

$$\left\{\frac{1}{2m^*}p_x^2+\frac{1}{2m^*}[p_y+eA(x)]^2\right\}\Psi(x,y)=E\Psi(x,y), \quad (2)$$

where $A(x)$ is the vector potential in the Landau gauge, $B(x)={\rm d}A(x)/{\rm d}x$. For convenience, we express all quantities in dimensionless units: (1) coordinate is in units of $l_{\rm E}$, (2) energy in units of $\hbar/m^*l_{\rm E}^2$, (3) magnetic field in units of $\hbar/el_{\rm E}^2$. Since the problem is translational invariant along the y direction, then $\Psi(x,y)={\rm e}^{{\rm i}qy}\psi(x)/\sqrt{L_y}$, where q and L_y are the wave-vector and structure length in the y direction, respectively. Substitution of $\Psi(x,y)$ into equations (1, 2) yields the following 1D dimensionless Schrödinger equations

$$\left[\frac{{\rm d}^2}{{\rm d}x^2}-2U(x)+2E-q^2\right]\psi(x)=0, \quad (3)$$

and

$$\left\{\frac{{\rm d}^2}{{\rm d}x^2}-[A(x)+q]^2+2E\right\}\psi(x)=0. \quad (4)$$

The function $V(x,q)=[A(x)+q]^2$ can be interpreted as an effective q-dependent electrostatic potential. From equations (3, 4), we find that: electron motion in ES is in fact a one-dimensional process, while electron tunneling in MS is inherently a complicated two-dimensional process, which depends on the electron's wave vectors in the longitudinal and transverse directions of the 2DEG.

Using the transfer-matrix method derived in reference [6], one can obtain the transmission coefficient for electron tunneling through the finite ES(MS):

$$T(E,q)=\{1+(T_{11}^2+T_{22}^2+k^2T_{12}^2+T_{21}^2/k^2-2)/4\}^{-1}, \quad (5)$$

where $T_{ij}(i,j=1,2)$ are the elements of the transfer matrix connecting the incident wave functions to the outgoing wave functions and $k=\sqrt{2E-q}$. In the ballistic regime, conductance can be derived as the electron flow averaged over half the Fermi surface [4–7]

$$G/G_0=\int_{-\pi/2}^{\pi/2}T(E_{\rm F},\sqrt{2E_{\rm F}}\sin\theta)\cos\theta\,{\rm d}\theta, \quad (6)$$

where θ is the angle between the incidence velocity and the x axis, $E_{\rm F}$ is the Fermi energy, $G_0=e^2m^*v_{\rm F}L_y/\hbar^2$, and $v_{\rm F}$ is the Fermi velocity of electrons.

We begin with the study of the resonant peak splitting of ballistic conductance for electron tunneling through the finite ES structure. In Figures 2a-c we present the conductance as a function of incident energy for three kinds of ES structure, one of which is the ES of identical barriers named AA ES and another two are the ES formed by arranging two different blocks A and B named AB ES and BA ES. The structure parameters in Figure 2 are chosen as $U_{\rm A}=3, U_{\rm B}=1.5$ and $l_{\rm EA}=l_{\rm EB}=l_{\rm E}=1$. It can be shown that the resonant peak splitting for ballistic conductance in the AA ES is $(n-1)$-fold (Fig. 2a), *i.e.*, the number of resonant peaks equals the number of ES barriers minus 1, while the $(n-1)$-fold resonant conductance peak splitting does not hold for the n-barrier ES made of two different-height barriers (Figs. 2b, c). However, an interesting phenomenon occurs: one resonant window (energy domain where the resonant peaks appear) for the ES of identical barriers splits into two subwindows for the ES formed by arranging two different-height barriers. Within each of the two subwindows, the number of resonant peaks is one less than the number m of the renormalized building blocks AB or BA, with which ES can be viewed as the identical-barrier ES, except for the BA ES of 5 barriers. Then we can say that the resonance split of ballistic conductance peaks in the ES made of two different-height barriers is $(m-1)$-fold. It should be pointed out that the single higher barrier A can be regarded as one renormalized block while the lower barrier B cannot. The reason is that the lower barrier has higher transmission and thus the additional quasi-bound state is difficult to form. For electron tunneling through the finite ES, the resonant peak splitting for ballistic conductance is the direct result of transmission split. It is known that resonance occurs, *i.e.*, the transmission coefficient is 1, as the tunneling part of the incident electron energy is equal to the energy of a quasi-bound state. Due to the tunneling coupling between wells through the barrier, the degenerate eigenlevels of the independent well will split. Theoretical and experimental investigations [6,8] have shown that the resonance split of transmission is

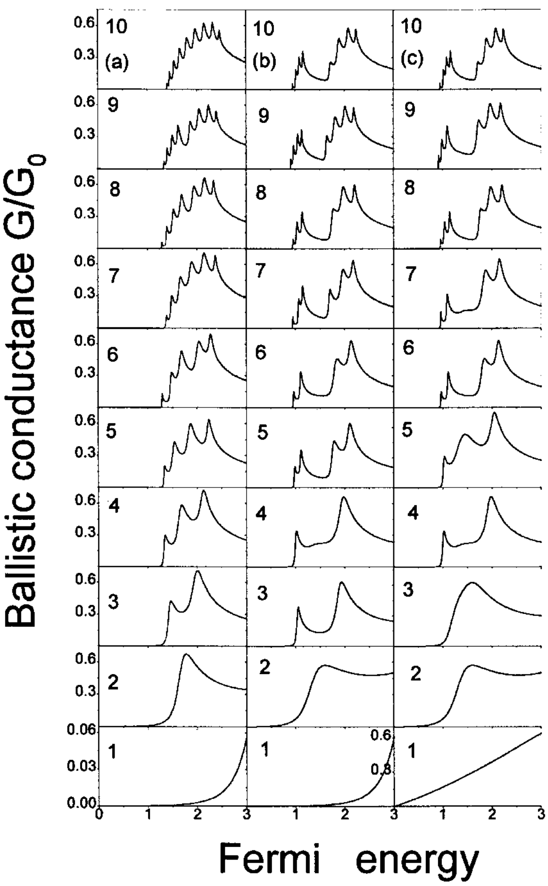

Fig. 2. Ballistic conductance *versus* energy for different number of electric barriers for (a) ES of identical A barriers, (b) ES of periodically arranged two-barrier structure AB and (c) ES of periodically arranged two-barrier structure BA. Here, $U_{\rm A} = 3, U_{\rm B} = 1.5$ and $l_{\rm EA} = l_{\rm EB} = l_{\rm E} = 1$.

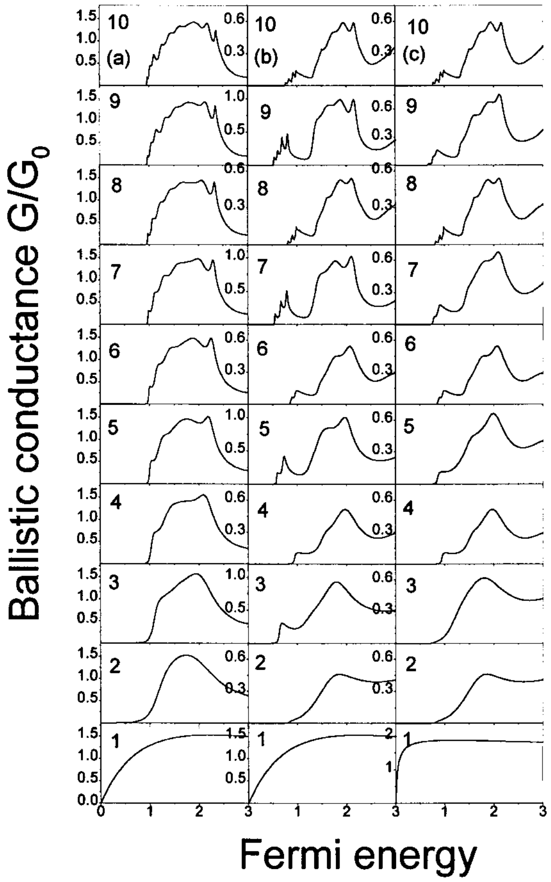

Fig. 3. Ballistic conductance *versus* energy for different number of magnetic barriers for (a) MS of identical A barriers, (b) MS of periodically arranged two-barrier structure AB and (c) MS of periodically arranged two-barrier structure BA. $B_{\rm A} = 3, B_{\rm B} = 1.5$ and $l_{\rm BA} = l_{\rm BB} = 1$.

$(n-1)$-fold for electron tunneling through the identical-n-barrier semiconductor superlattices. For electron tunneling through the n-barrier ES formed by periodically arranging two different barriers the $(n-1)$-fold resonant splitting of transmission peaks no longer holds since the coupling between the wells are different. However, if one regards the two-well structure with two low-transmission barriers and one high-transmission barrier as a single well in which there exists two well-separated quasi-bound levels [9], then two quasi-bound levels will split into two corresponding well-separated subwindows within each the number of quasi-bound levels is closely related to the number of low-transmission barriers. Therefore in each of the split resonant windows, the resonant peak splitting for transmission in the ES formed from two different barriers is $(m-1)$-fold, where m is the number of low-transmission barriers, which is also the number of the renormalized building blocks. Though the resonant transmission peaks have different positions for different transverse wave vector q, the number of resonant transmission peaks is the same for different q. According to equation (6), ballistic conductance is the transmission averaged over all possible transverse wave vectors, therefore the number of observable resonant conductance peaks is the same as for transmission. Hence, the resonant peak splitting for ballistic conductance is the same as for transmission in ES structures.

Now we inspect the resonance split of ballistic conductance peaks for MS. Calculated conductances for three kinds of MS structures AA MS, AB MS and BA MS are given in Figures 3a-c, respectively. In Figure 3 the parameters are chosen as $B_{\rm A} = 3, B_{\rm B} = 1.5$ and $l_{\rm BA} = l_{\rm BB} = 1$. By comparing the number of resonant conductance peaks with that of magnetic barriers in the corresponding MS, one can find the same resonant splitting rule for ballistic conductance peaks as that in the ES case, *i.e.*, for electron tunneling through an identical-n-magnetic barrier MS, the resonant conductance peak splitting is $(n-1)$-fold; while for MS made of two different magnetic barriers, one resonant window in the former splits into two subwindows, within each of which the resonant splitting is $(m-1)$-fold, where m is the number of the renormalized building magnetic barrier blocks. In MS structure, due to the close relation between transmission and transverse wave vector q, there exists no explicit unified rule for resonance splitting of transmission peaks. Nevertheless, since ballistic conductance is derived as the transmission over half the Fermi surface, ballistic conductance can be viewed as the transmission of the electron's collective tunneling with a characteristic positive transverse wave vector q through an averaged effective potential $V_{\rm ave}(x)$, which has the same number of barriers as the magnetic barriers of the MS profile [7]. Then the number of resonant conductance peaks for MS is also the same as that of resonant transmission peaks for electron tunneling through the ES structure which has similar potential structure. Therefore, for electron tunneling through the finite MS structure, the rule for the resonance split of ballistic conductance peaks is the same as that for the corresponding ES structure.

What would result if electric and magnetic modulations with the same period were simultaneously applied? It is shown in references [10,11] that, as long as the electric and magnetic modulation are in phase, the resonant peak splitting of ballistic conductance does not change. If they were out of phase, the situation would be significantly different. In this case, resonant splitting for ballistic conductance peaks would depend on the strength of the electric and magnetic modulations. Very recently, Ibrahim and Peeters [12] have studied magnetic superlattices comprising infinite magnetic barriers and diffusive conductance in the framework of semiclassical theory. They found that magnetic minibands have a pronounced effect on the oscillatory longitudinal conductivity and trivial influence on the transverse conductivity. In contrast, the magnetic superlattice we considered is finite and the conductance studied is ballistic. Instead of magnetic miniband structure, in the energy spectrum of a finite MS, there only exist discrete quasi-bound levels through which the tunneling is resonant and thus transmission and conductance exhibit resonant structure.

In summary, we studied the resonant peak splitting for ballistic conductance in electric and magnetic superlattices. Our results showed that the resonance split of conductance peaks is the same for electric and magnetic superlattices. For electron tunneling through the finite ES (MS) of periodically arranged identical n electric (magnetic) barriers, the resonant peak splitting for ballistic conductance is $(n-1)$-fold. However, for the ES (MS) constructed by periodically juxtaposing two different electric (magnetic) barriers, one resonant window in the former structure splits into two subwindows in the latter structure, within each of which the resonant peak splitting is $(m-1)$-fold, where m is the number of the renormalized electric (magnetic) barrier buildings blocks, with which the ES (MS) can be regarded as the ES (MS) of periodically arranged identical m barriers.

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